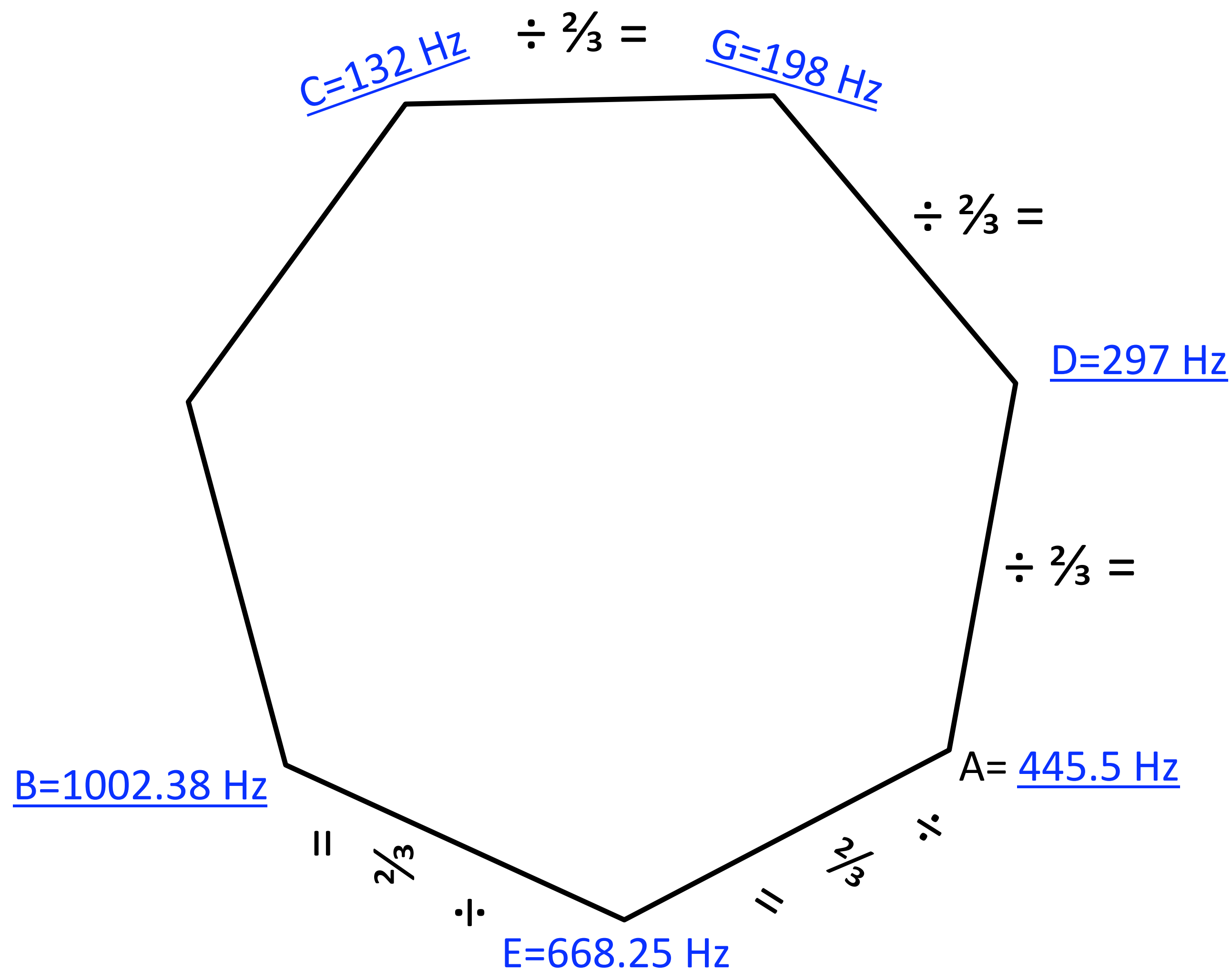
Here are the first steps to finding vales in Hz for a Pythagorean scale:



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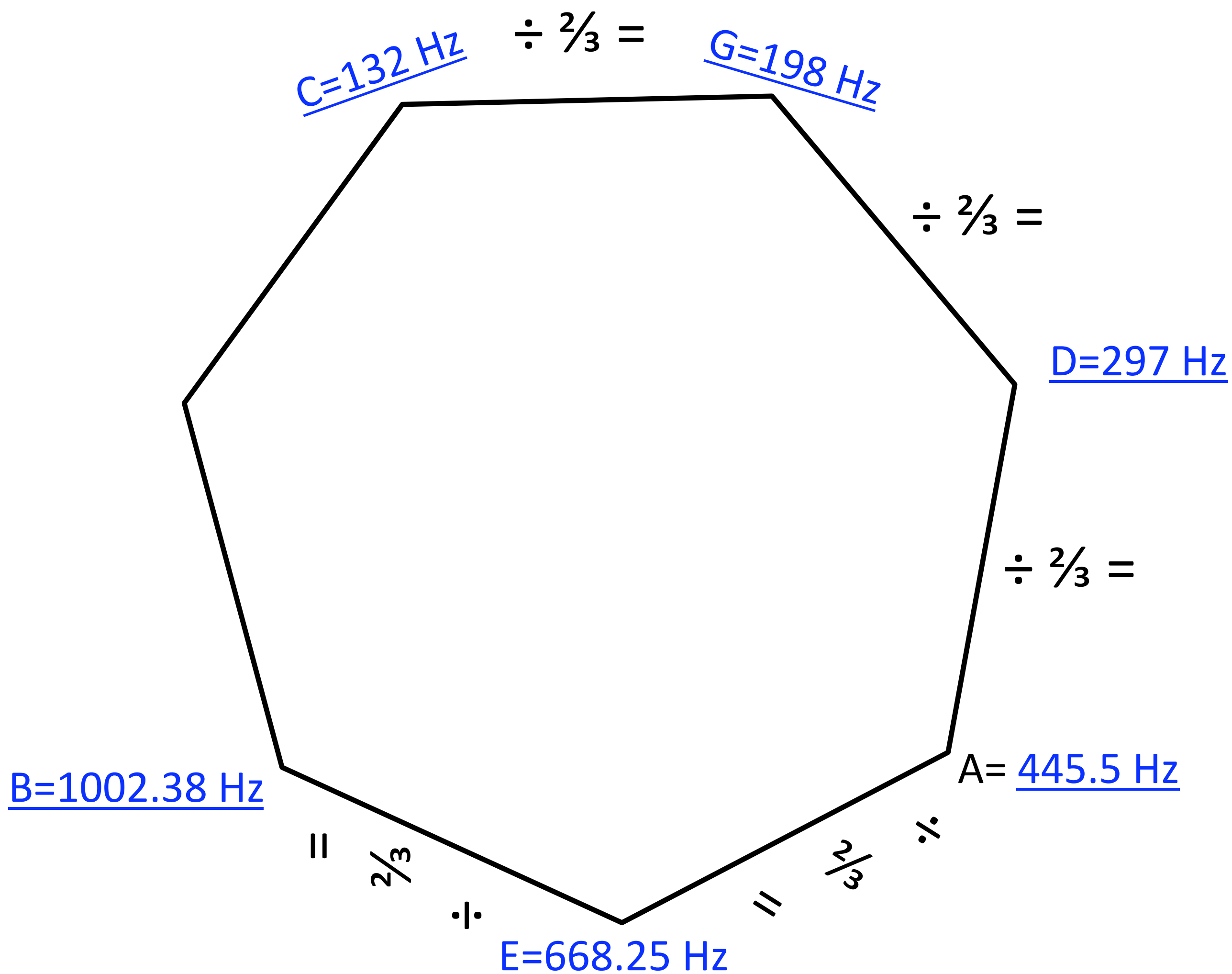
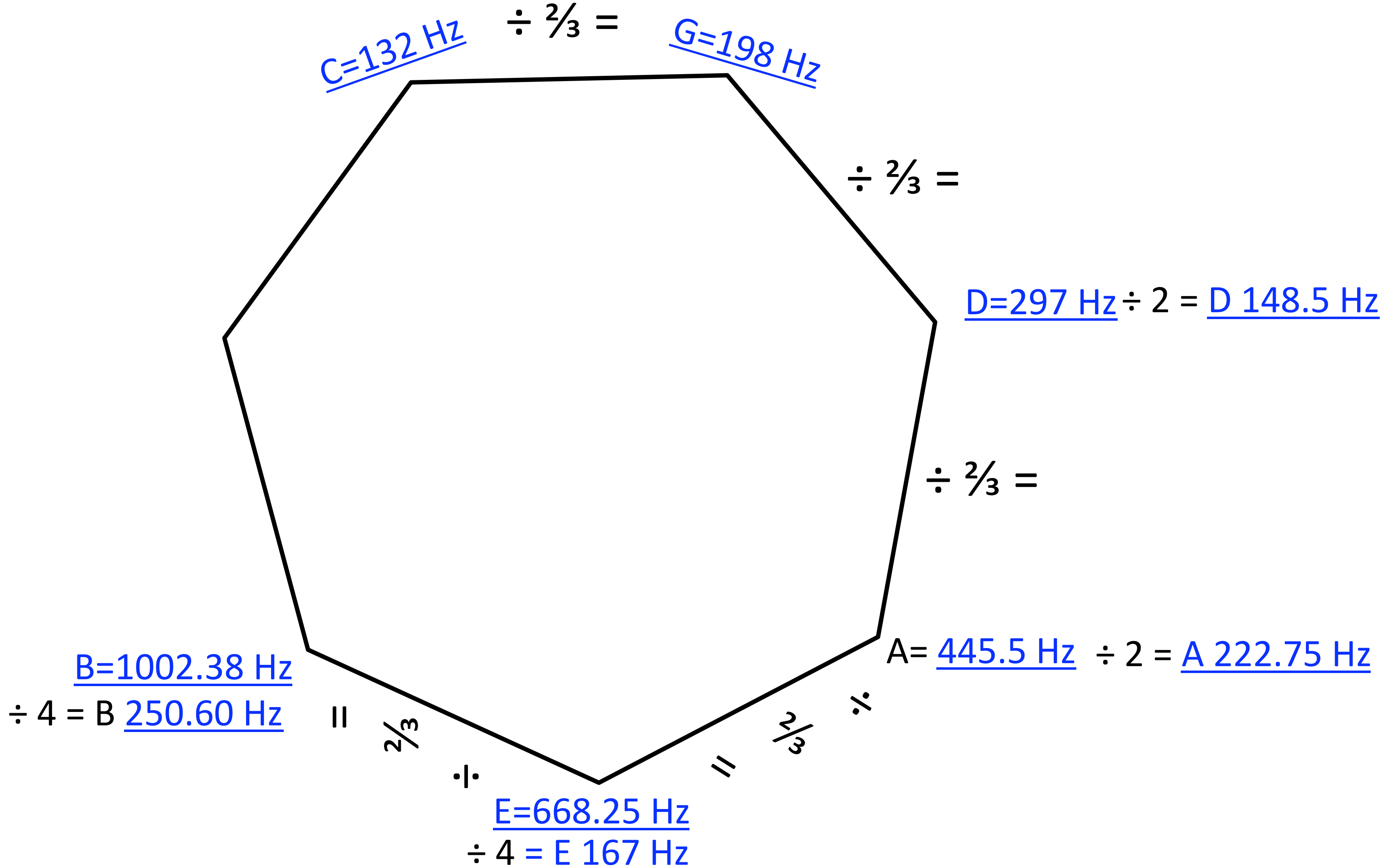


The resulting series of just fifths sounds like this

Why doesn’t this sound like a scale?

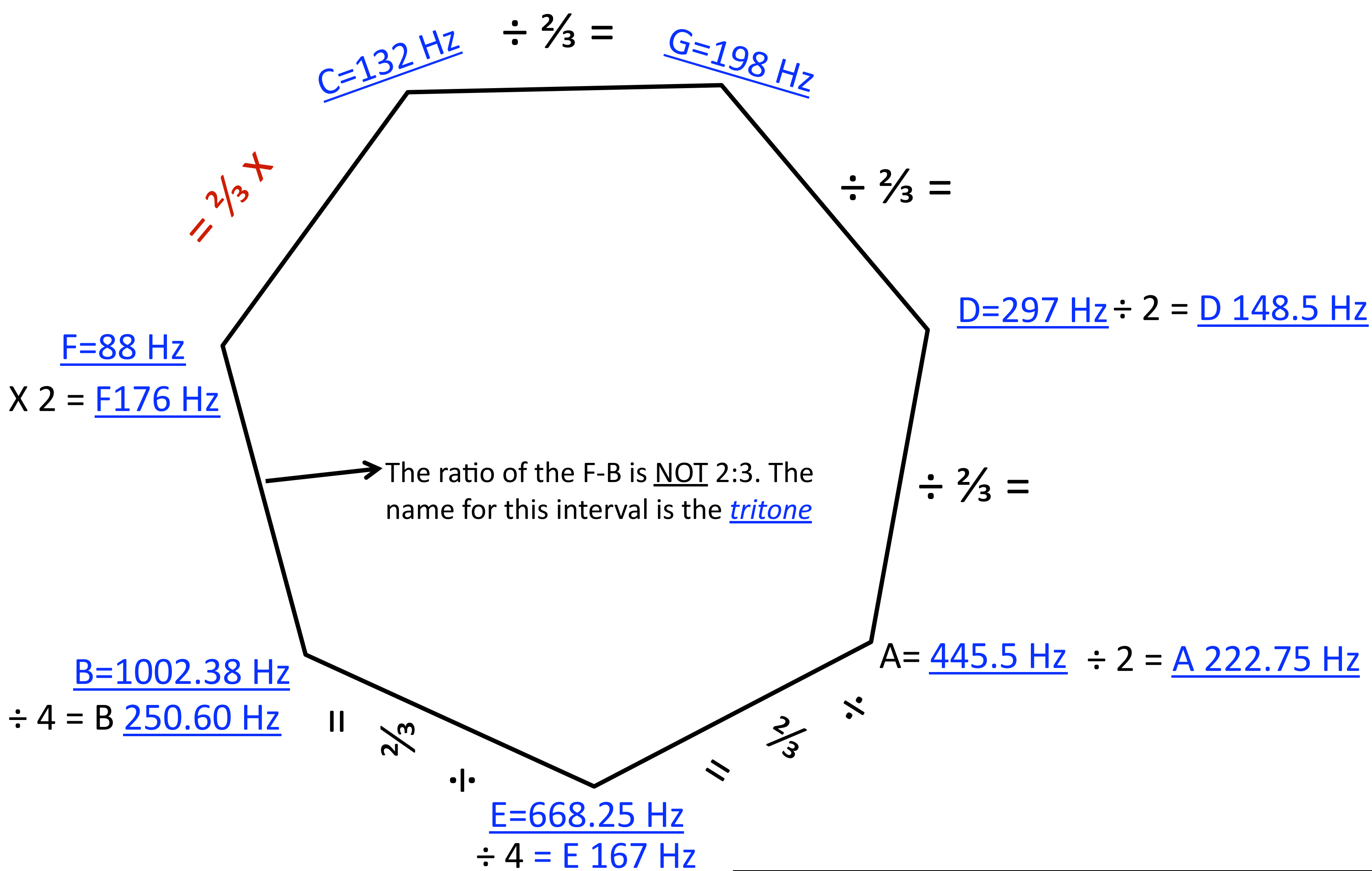
To derive a scale we have to fit these pitches within a singe 2:1 interval. The value of every C in Hz = 132 x 2^n (octave equivalency), and each pitch can be fit within a 2:1 interval by dividing that pitch's frequency in Hz by a factor of 2 :



The new scale sounds like this

What's Missing?

The pitch we will call F is missing. To get an F we must find a pitch *below* C which is related to C by ⅔, then double its frequency to fit in within an octave:

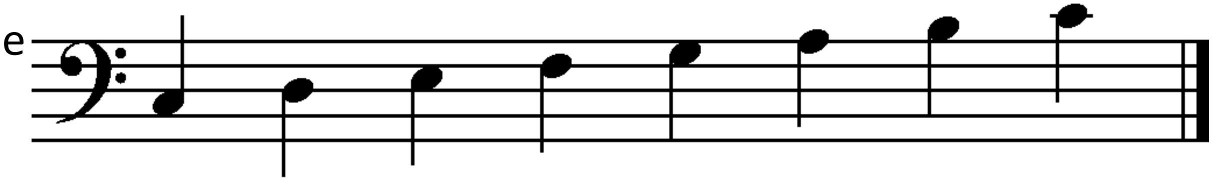


The complete scale sounds like this

Congratulations! We built a Pythagorean major scale using simple math

This Pythagorean scale is beautiful and useful for lots of music. All the 2:3 relationships are just with no beats.

Pythagorean scale within an octave:



C132hz D148.5Hz E167Hz F 176Hz G198Hz A222.75Hz B 250.6Hz C264Hz

13th century religious chant: lots of 2:3 intervals

Notice how the 2:3 ratio has a very powerful sound. Also notice it has two *inversions*; the perfect fifth (from C-G) and the perfect fourth (from G-C). The fifth and fourth are awesome and Pythagorean tuning focuses our attention on those sounds.

Starting in the 15th Century, musicians in Europe became interested in the 4:5 interval, which is called a major third. In pythagorean tuning. C-E is an example of a major third

Played together with a pure fifth, the pure major third interval forms a pure *triad*. This chord, which can be given the ratio 4:5:6, became the basis of the European musical system.

In Pythagorean tuning the triads are not pure. The C-E ratio is 64:81. The C to G ratio is pure 2:3 but when the major third is added a Pythagorean triad has beats.

These out of tune Pythagorean triads were not acceptable for musicians and tuners. They began to explore both the properties of 4:5:6 triads and how to tune them.

A new scale with pure thirds

Remember we began building our Pythagorean scale using a fundamental pitch. As musicians began exploring pure triads they began to develop an affinity for pure triads based on the fundamental pitch (C), fifth scale degree (G), and fourth scale degree (F):

C triad (I) G triad (V) F triad (IV)

The interval between C-G= 2:3. The interval between F-C= 2:3. Triads built on these pitches form a relationship which musicians enjoyed exploring, and these relationships are still familiar to us.

They are referred to as I, V, and IV chords, or tonic, dominant, and subdominant. They form the basis for the blues.

Here's some Mozart. This music is based on triads. At :42 the piece moves from I to V.

Here's the move from I to V.

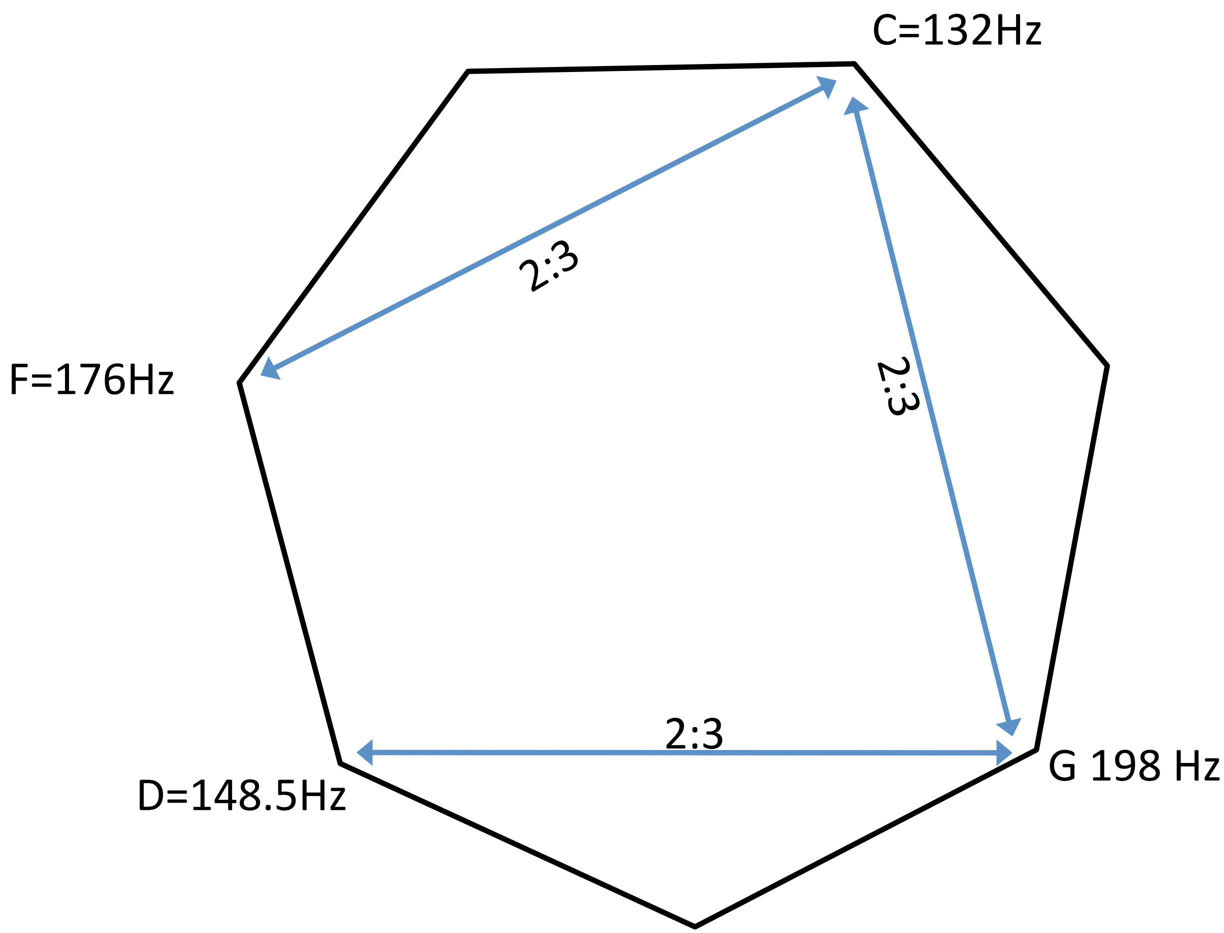
Here's some Mozart. This is what a move to IV would sound like.

Here's the whole movement again moving to the IV.

This interest in triads leads to a new way of tuning the major diatonic scale where the goal is to create a scale where the triads based on C, F, and G are all pure. Also, F-C, and C-G will be just fifths.

Tuning with pure triads

For this tuning we will use a circle of thirds, not fifths, to represent the octave. As in Pythagorean tuning, C is the fundamental, G=⅔ C, D= ⅔ G, and F = 3/2 C.



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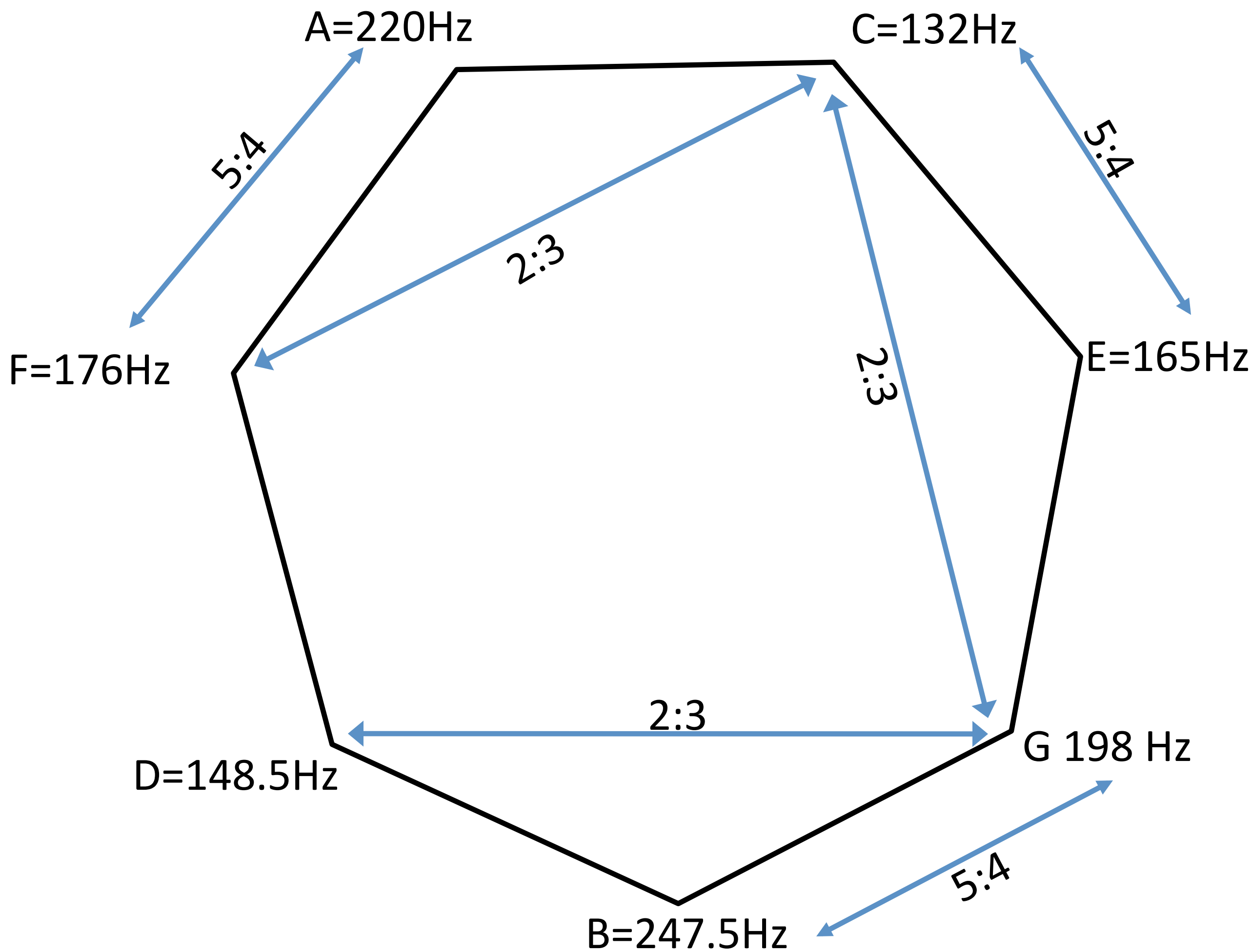

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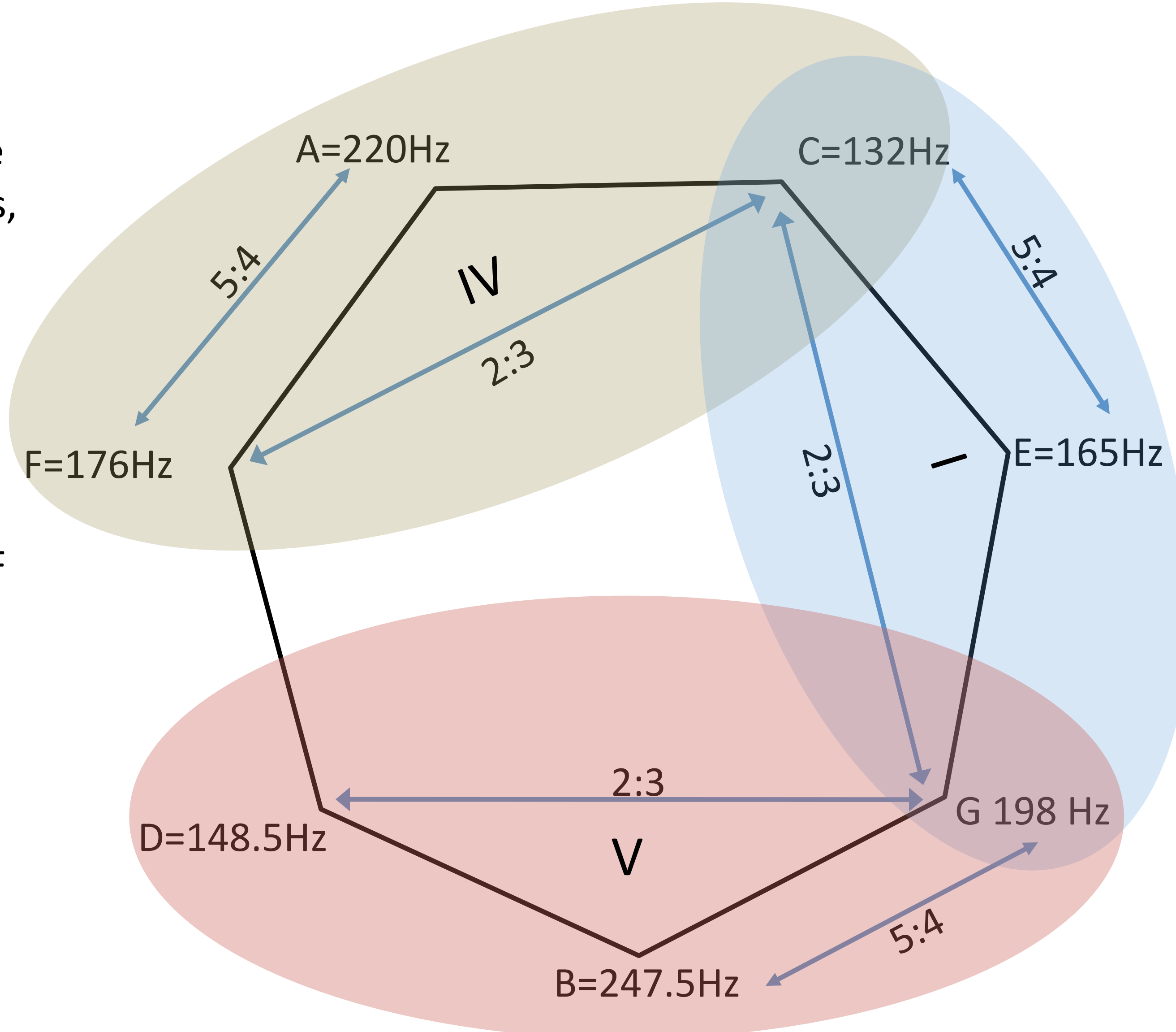

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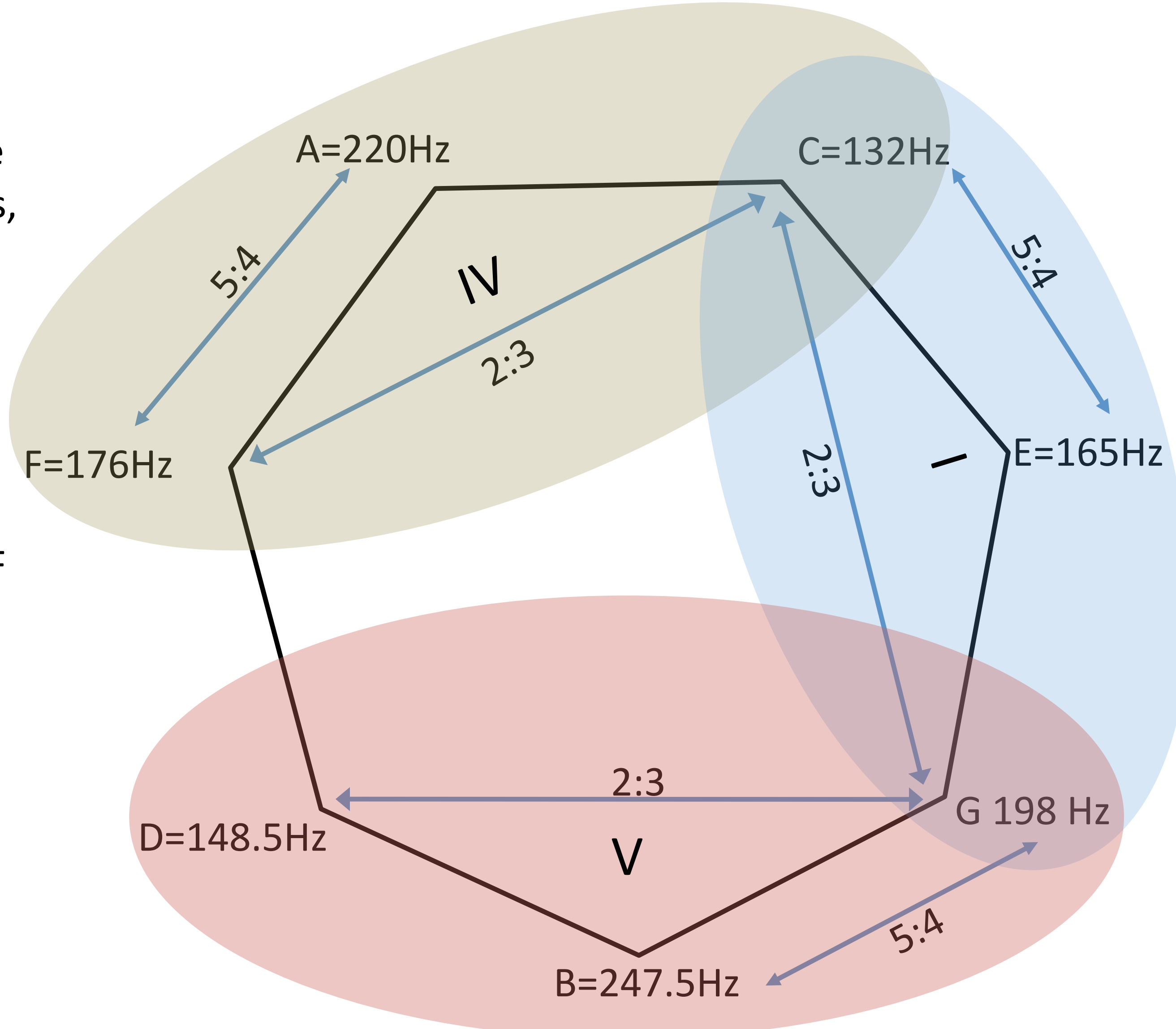
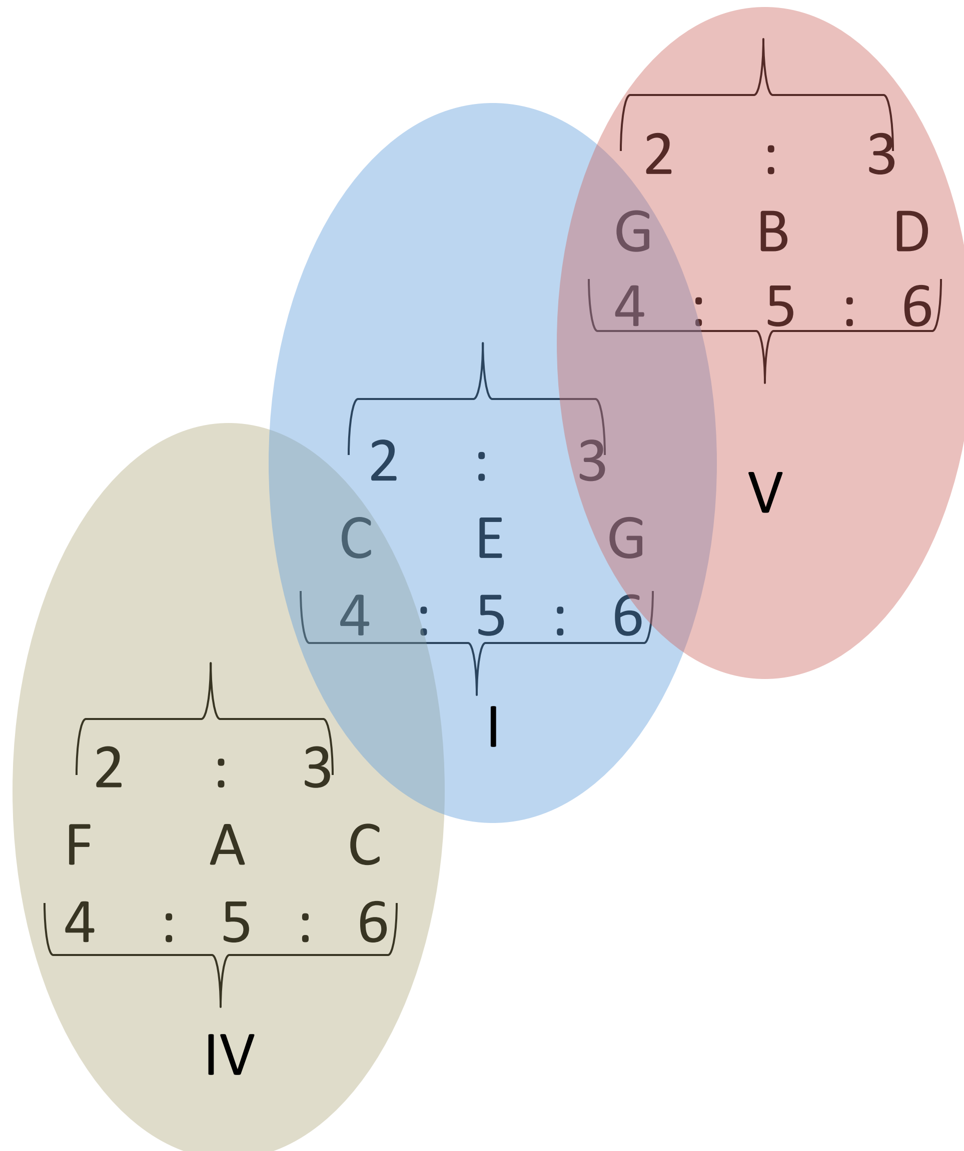


Scale with pure triads (deviation from Pythagorean tuning in red):

Tuning with pure triads

Here is another way to think about tuning with pure triads. C is the fundamental pitch, and the C triad becomes the *key* center of this scale. Other harmonies, especially the F and G triad, are heard relative to C.



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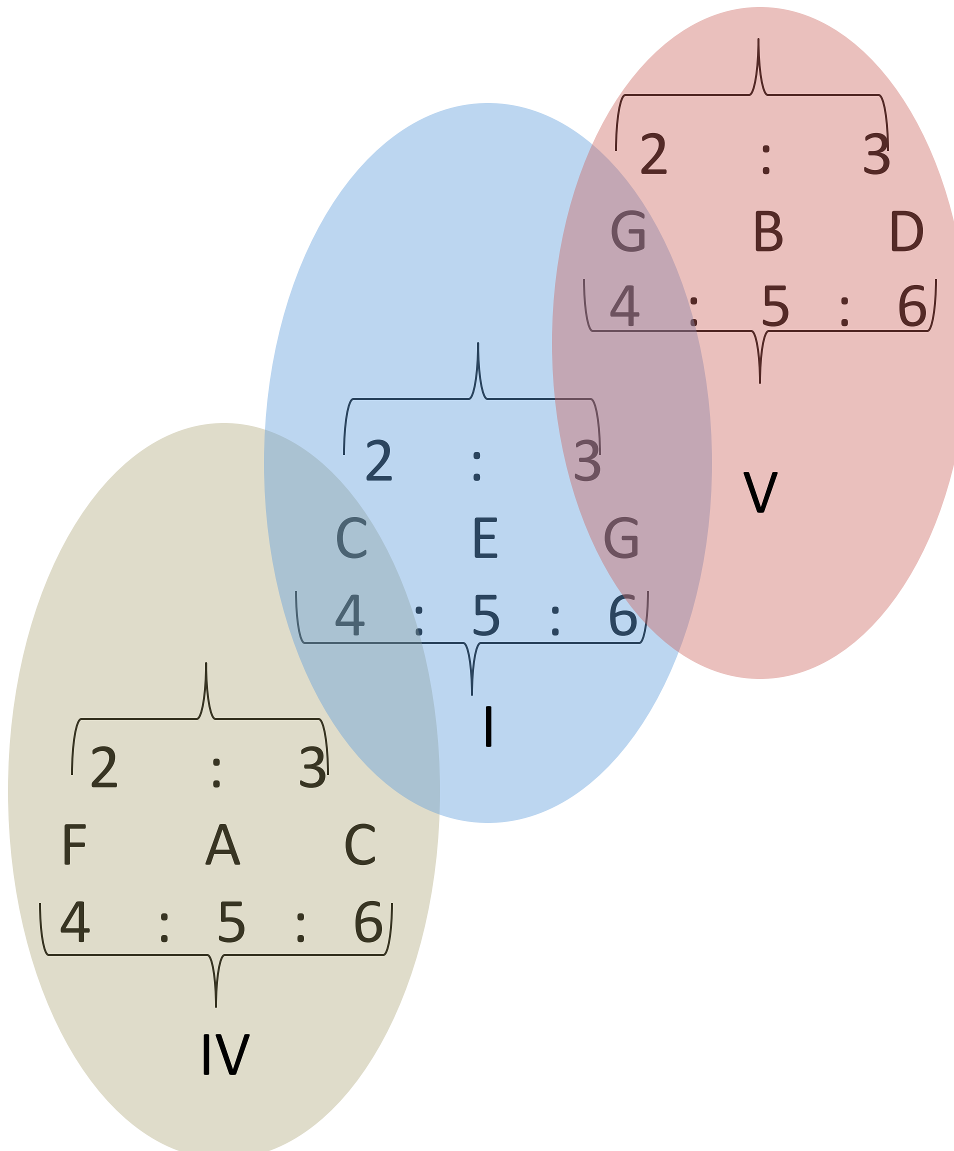
As this tuning system became more common, musicians wanted to expand it to include a new scale based on A. This scale became known as the relative minor, and moving from a major key to its relative minor is a familiar sound. Yesterday



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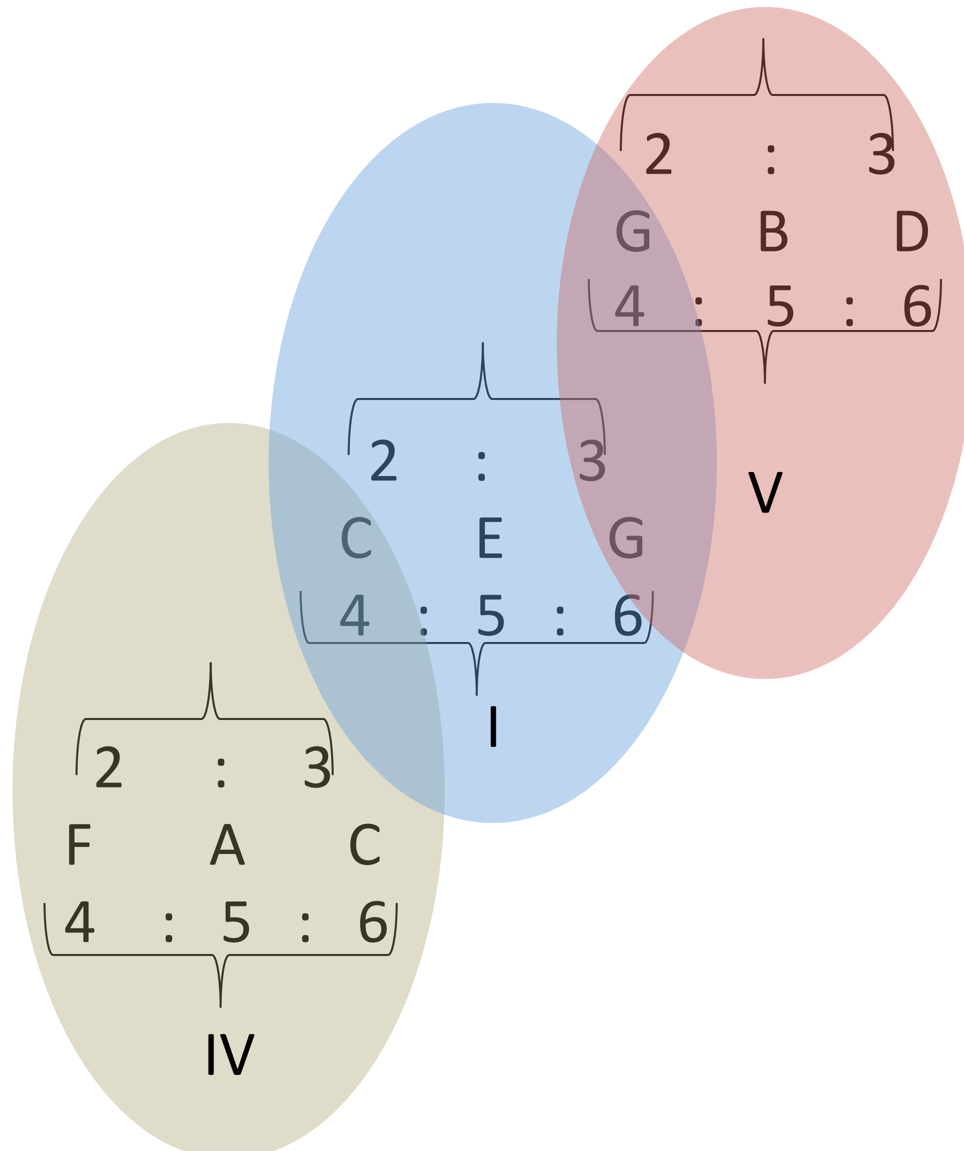


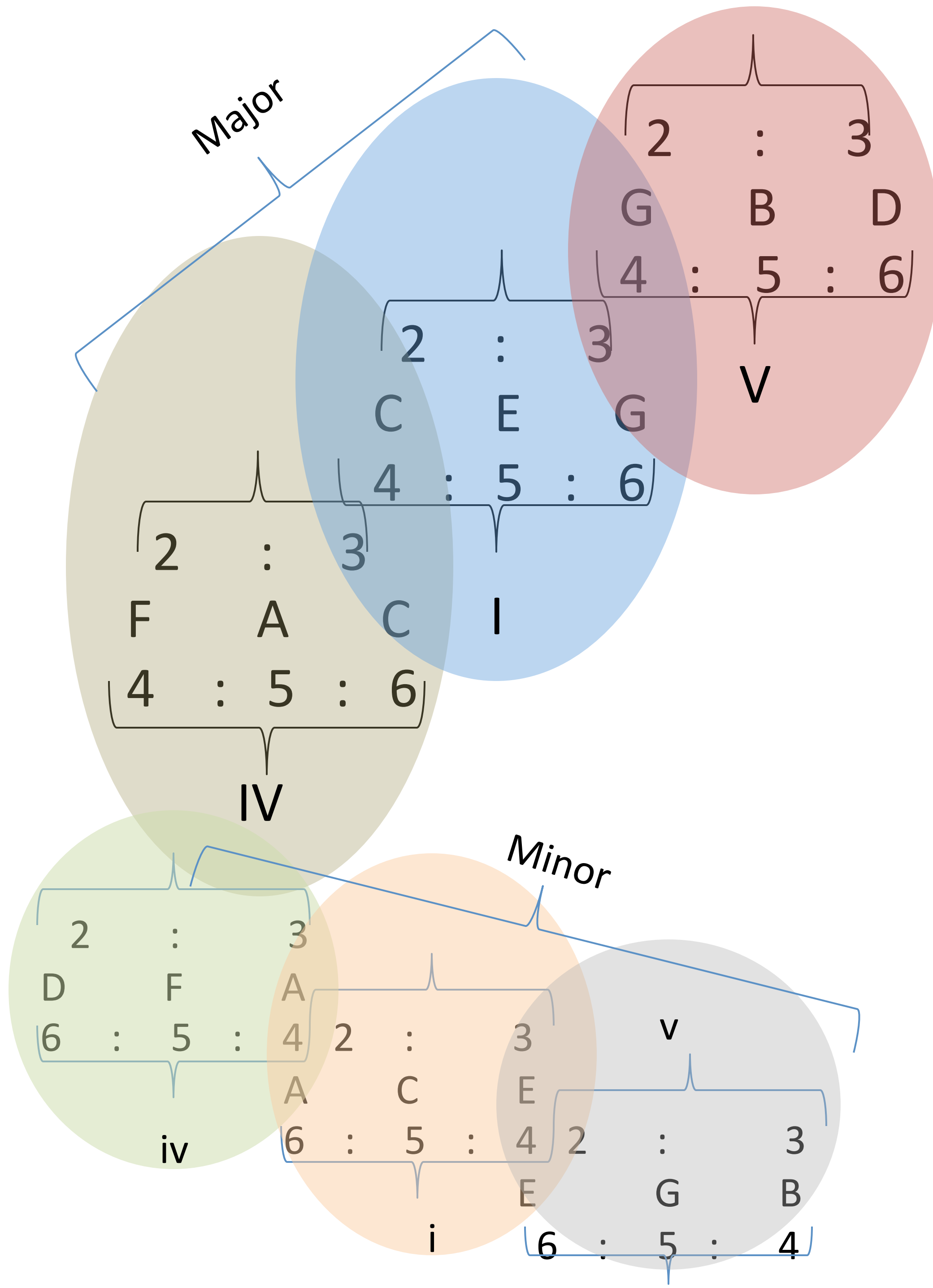
This relative minor scale based on A was very pleasing to the musicians of the 16 and 17 Centuries. They wanted to develop a tuning that created a relative minor scale whose i, iv, and v pitches were in a 2:3 ratio , and where the minor triads based on i, iv, and v were pure.

Minor and Major

We know the pure major triad has a 4:5:6 ratio. The 4:5 ratio is the major third.

The pure minor triad has a 6:5:4 ratio. It is reciprocal to a major triad. The 6:5 interval is called a minor third.

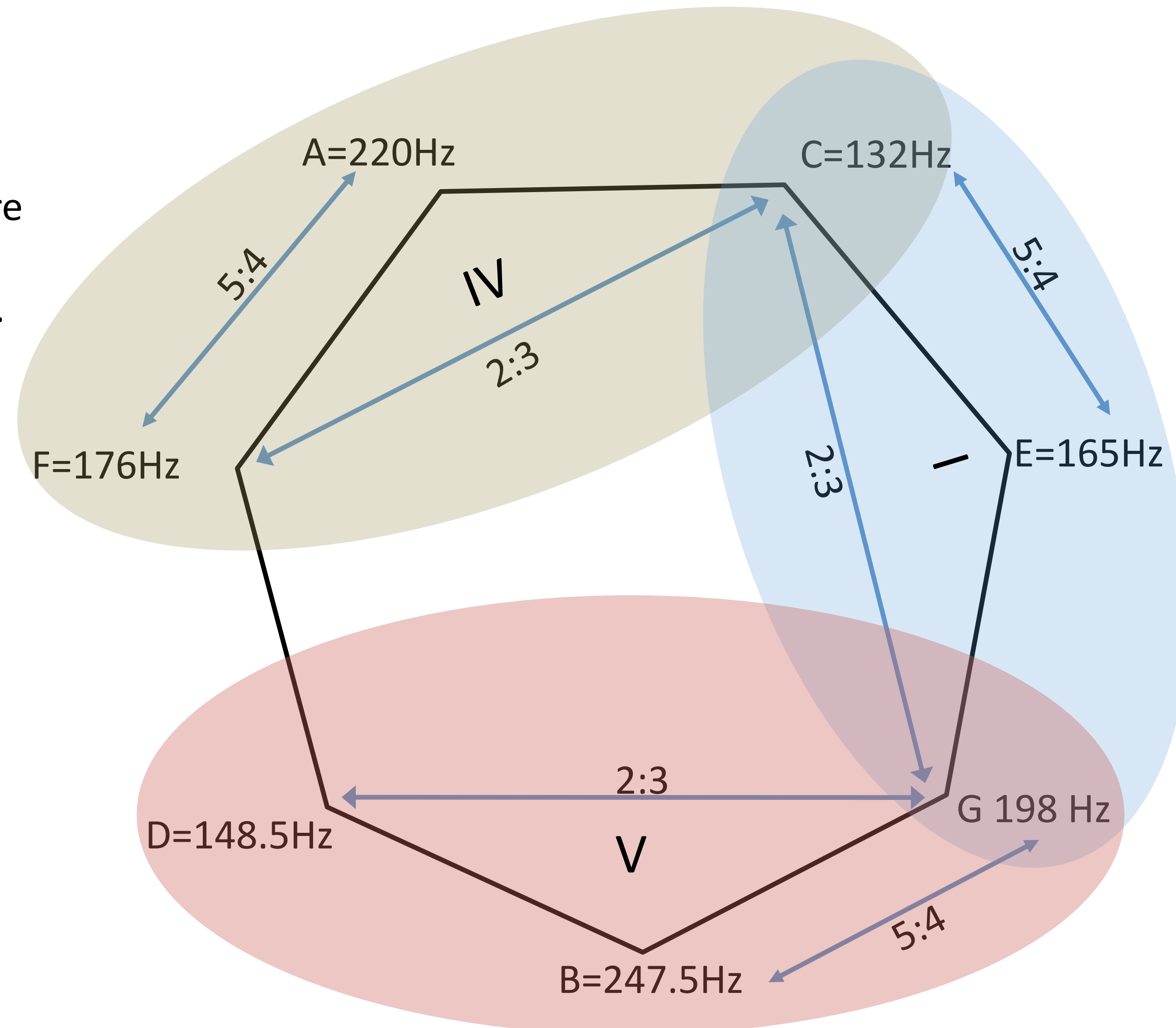
A scale with pure major triads on C, F, and G and pure minor triads on a, d, and e looks like this:



What happens when we try to create this scale?

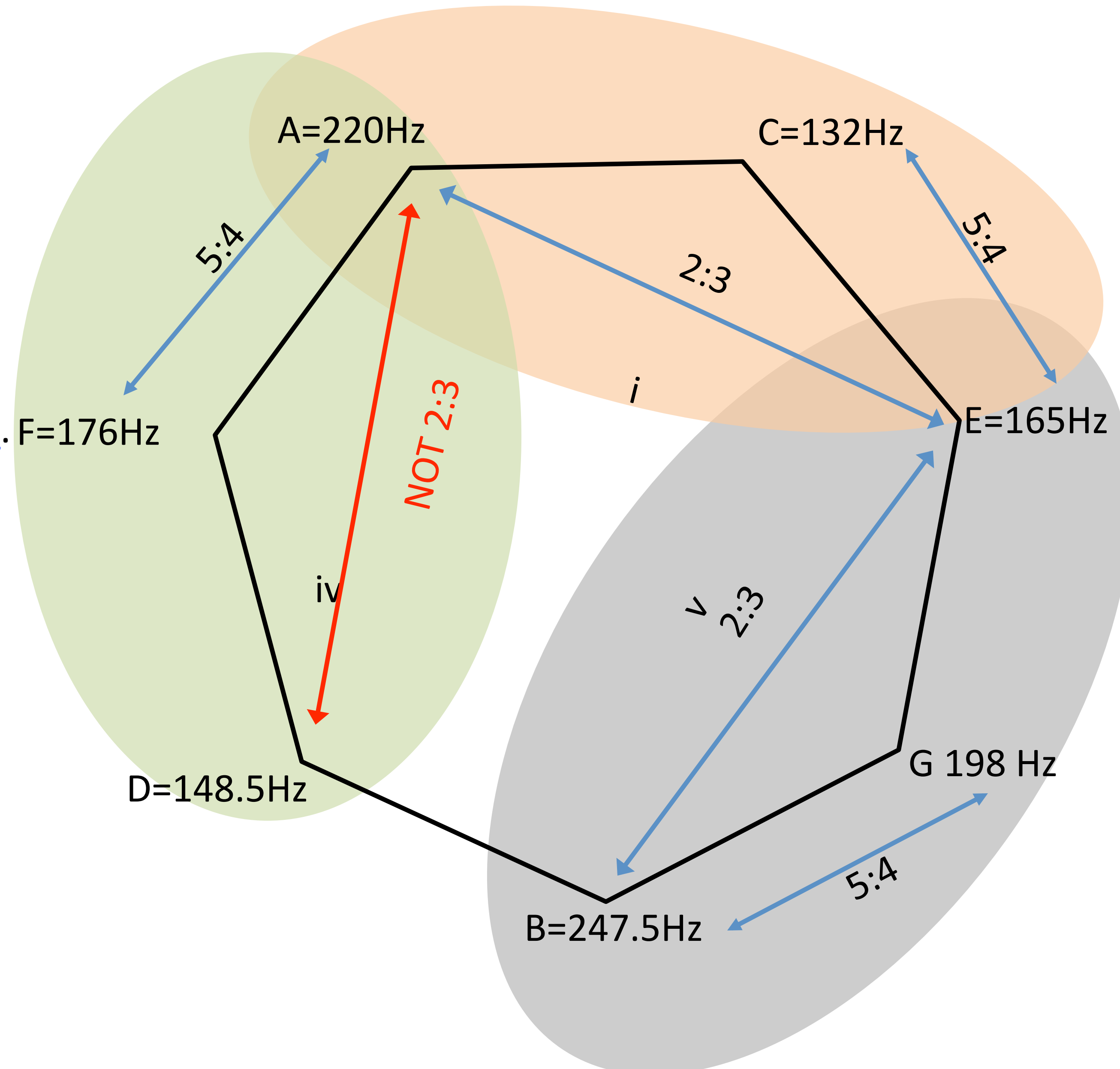
Tuning with pure triads:
The comma

Here is our scale with pure thirds showing the three major triads I IV V circled.



Tuning with pure triads:
The comma

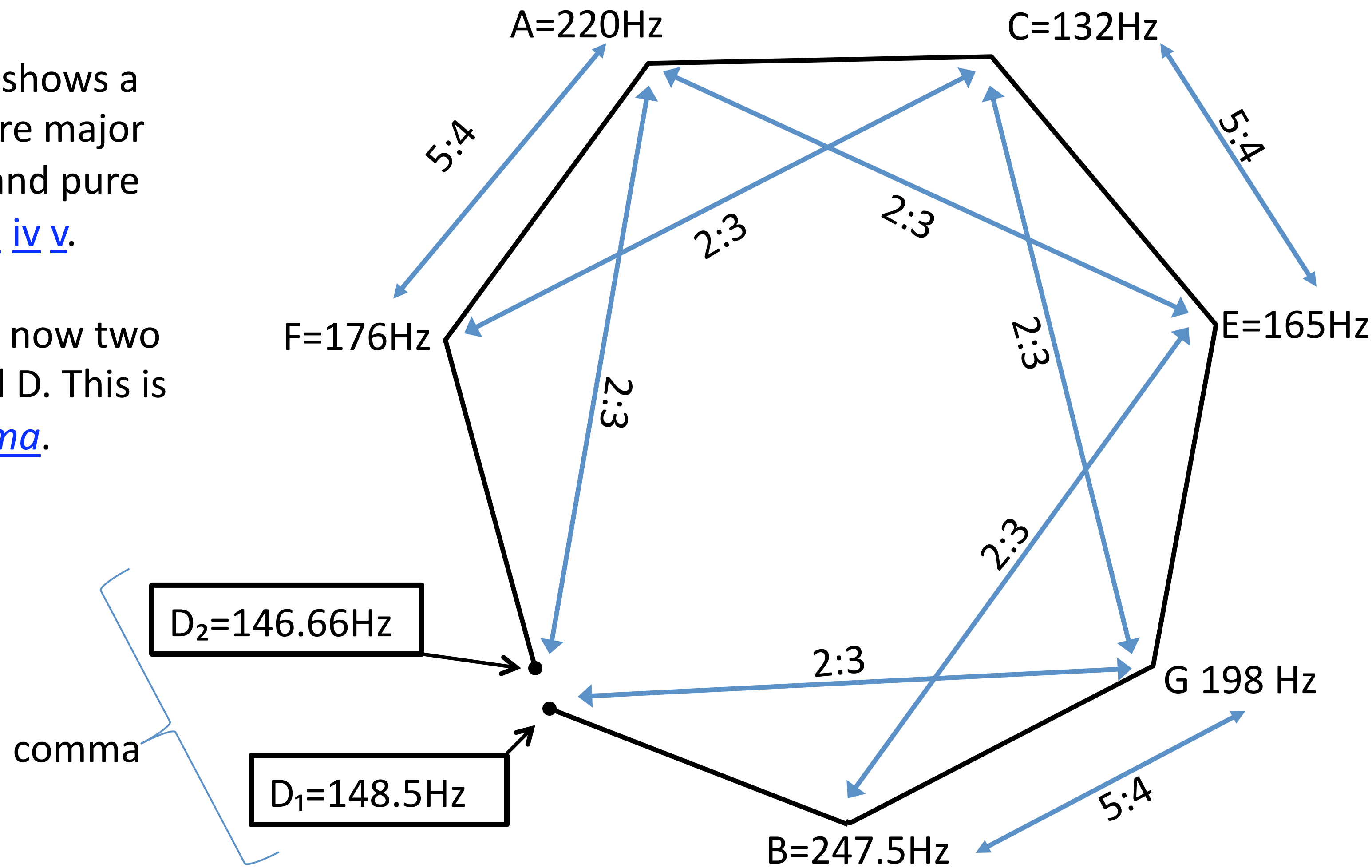
Now we have circled the three minor triads i iv v. The i triad is pure 6:5:4, The v triad is pure 6:5:4, But the iv triad is not pure. In this tuning A:D is 27:40, a complex impure ratio. In order to make a pure 6:5:4 iv triad we need to create another pitch, D 146.66Hz, which is 2:3 A.



Tuning with pure triads:
The comma

This diagram shows a scale with pure major triads I IV V, and pure minor triads i iv v.

But there are now two pitches called D. This is called a *comma*.



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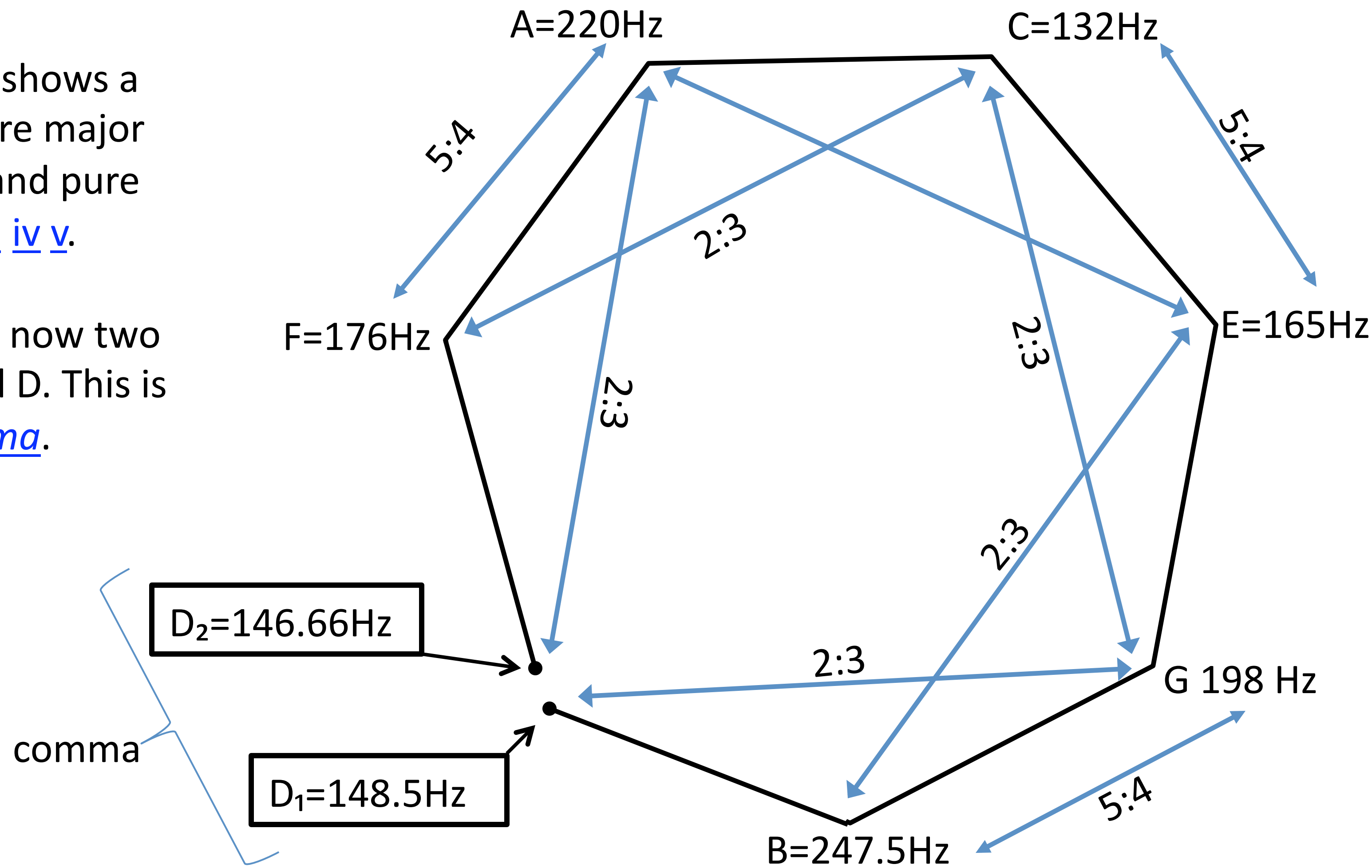

The comma keeps us from tuning a diatonic scale with pure 4:5:6 major triads and pure 6:5:4 minor triads.